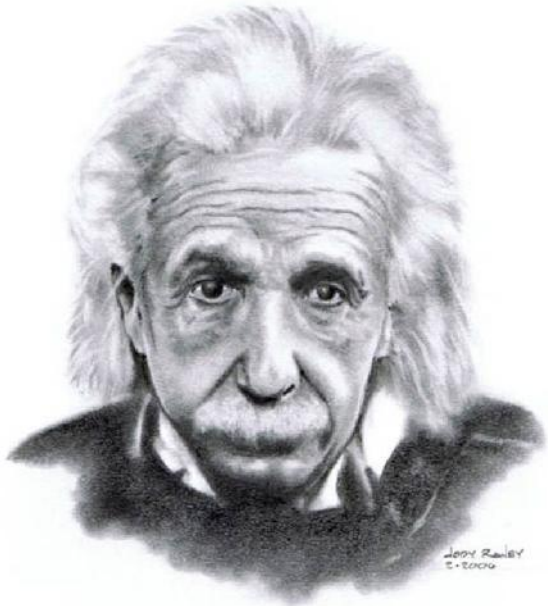
Elements of Relativity

? At Brookhaven National Laboratory in New York, atomic nuclei are accelerated to 99.995% of the ultimate speed limit of the universe—the speed of light. Is there also an upper limit on the *kinetic energy* of a particle?



From Sears & Zemansky's University Physics,
Vol. 1, 13th Edition 13th Edition
ISBN-13: 978-0321733382
ISBN-10: 032173338X

1905 Albert Einstein published three papers of extraordinary importance:

- 1st : an analysis of Brownian motion
- 2nd : (awarded the Nobel Prize) was on the photoelectric effect.
- 3rd : **special theory of relativity,** drastic revisions in the Newtonian concepts of space and time.

The special theory of relativity

based on just two simple postulates

- ❑ **Postulate 1:** the laws of physics are the same in all inertial frames of reference;
- ❑ **Postulate 2:** the speed of light in vacuum is the same in all inertial frames.

Implications

(1) Events that are simultaneous for one observer may not be simultaneous for another.

(2) When two observers moving relative to each other measure a time interval or a length, they may not get the same results.

(3) For the conservation principles for momentum and energy to be valid in all inertial systems, Newton's second law and the equations for momentum and kinetic energy have to be revised.

Relativity has important consequences in *all* areas of physics, including electromagnetism, atomic and nuclear physics, and high-energy physics.

1. Invariance of Physical laws

EINSTEIN'S FIRST POSTULATE

Principle of relativity: The laws of physics are the same in every inertial frame of reference.

An **inertial reference frame** is one where Newton's First Law, the law of inertia, holds. That means that if two reference frames are moving relative to one another at a constant velocity, the laws of physics in one are the same as in the other.

Example 1

What you see from the train   What your friend sees from the platform

- from a passenger's point of view It looks that the *station is moving backward*
- for someone standing on the platform it looks as if the *train is moving forward*.

It's just as correct to say that : *the train is still and the Earth is moving* as it is to say that *the Earth is still and the train is moving*.

Any inertial reference frame is as good as any other.

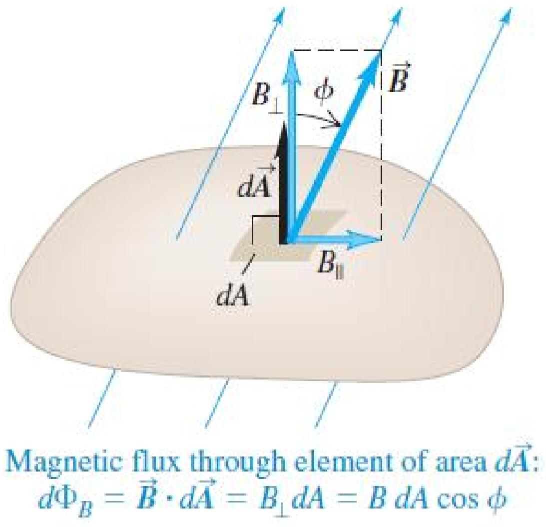
Example 2 the electromotive force (emf) induced in a coil of wire by a nearby moving permanent magnet

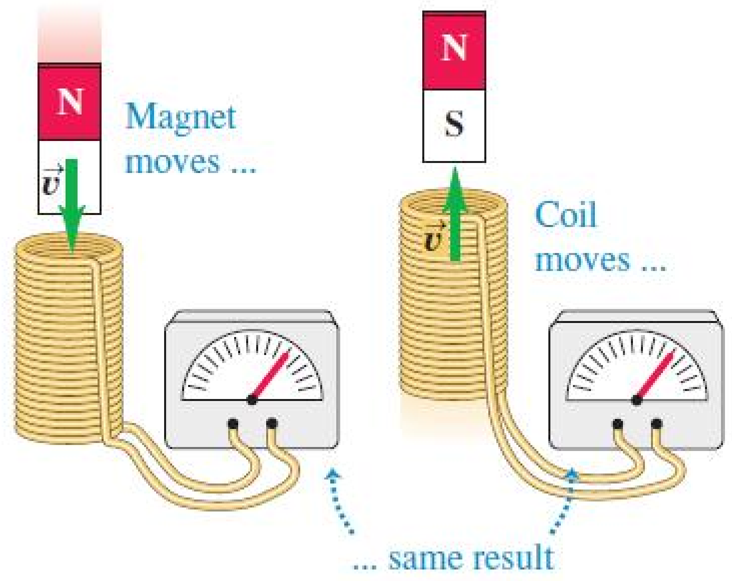
$$£ = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(\oiint_A \vec{B}\cdot d\vec{A}\right)$$

The same emf is induced in the coil whether

(a) the magnet moves relative to the coil or

(b) (b) the coil moves relative to the magnet.





all of the laws of electromagnetism are the same in every inertial frame of reference.

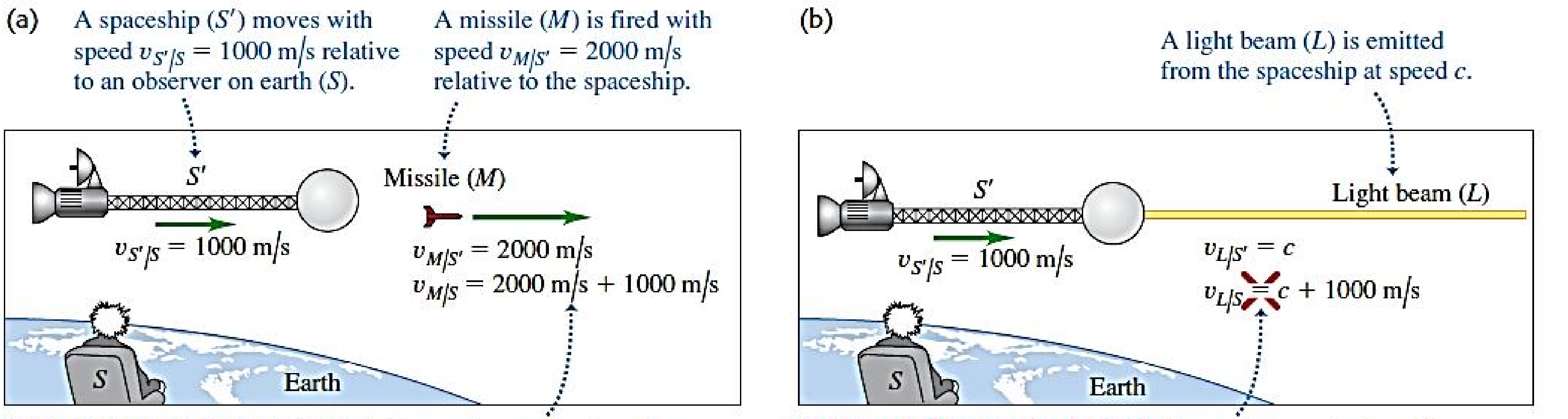
EINSTEIN'S SECOND POSTULATE

The speed of light in vacuum is the same in all inertial frames of reference and is independent of the motion of the source.

That is the speed of light c (~3x10^8 m/s) and it is independent on the motion of the source

(a) Newtonian mechanics makes correct predictions about relatively slow-moving objects;
(b) it makes incorrect predictions about the behavior of light.

Thinking experiment



NEWTONIAN MECHANICS HOLDS: Newtonian mechanics tells us correctly that the missile moves with speed $v_{M/S} = 3000$ m/s relative to the observer on earth.

NEWTONIAN MECHANICS FAILS: Newtonian mechanics tells us *incorrectly* that the light moves at a speed greater than c relative to the observer on earth ... which would contradict Einstein's second postulate.

This result contradicts our elementary notion of relative velocities, and
… it may not appear to agree with "common sense"

The Ultimate Speed Limit

Einstein's second postulate immediately implies the following result:

It is impossible for an inertial observer to travel at *c*, the speed of light in vacuum.

Einstein: "What would I see if I were traveling at the speed of light?"

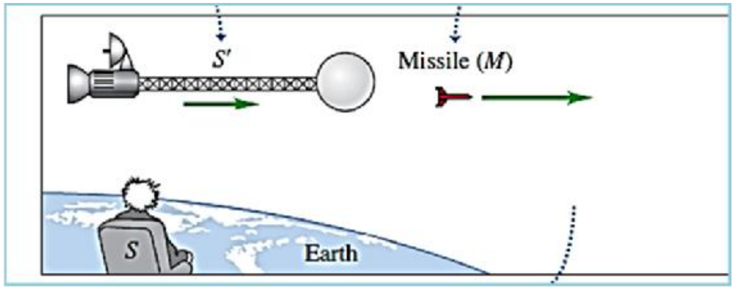
PROBLEM…you could not travel at c …

THE GALILEAN COORDINATE TRANSFORMATION

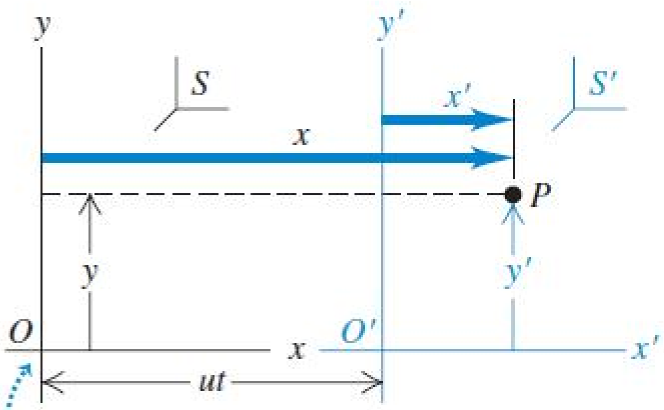
2 inertial frames of reference, labeled

S for the observer on earth (fix)

S' for the moving spacecraft



Frame S' moves relative to frame S with constant velocity u along the common x-x'-axis.



Origins O and O' coincide at time $t = 0 = t'$.

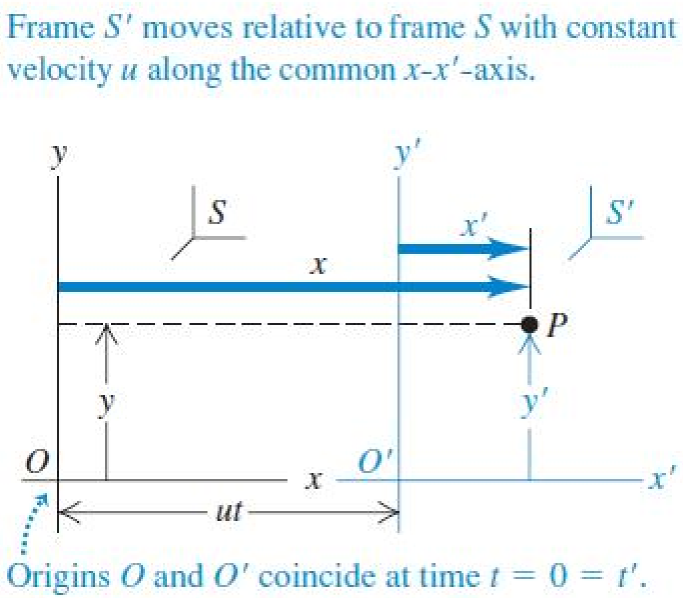
How we describe the motion of a particle P ?

…this can be exploratory vehicle launched from the spacecraft or a pulse of light from a laser

P Position described by coordinates

(x, y, z) in S

(x', y', z') in S'



P Position described by <u>coordinates</u>
(x,y,z) in S
(x’, y’, z’) in S’

fig ⇒

$$\begin{cases} x = x' + ut \\ y = y' \\ z = z' \end{cases}$$

Galilean Transformation
Based on familiar Newton laws of space an time

The measured time t and t’ are the same
(e.g. clock set on fix system)

<u>Velocities</u>

$$\frac{dx}{dt} = \frac{dx'}{dt} + u \quad \Rightarrow \quad v_x = v'_x + u \qquad \text{(Galilean velocity transformation)}$$

<u>Accelerations</u>

$$\frac{dv_x}{dt} = \frac{dv_x'}{dt} + 0 \Rightarrow a_x = a'_x \quad \Rightarrow \quad m\vec{a} = \sum \vec{F}_i \Rightarrow m\vec{a}' = m\vec{a} = \sum \vec{F}_i$$

Newton’s eqn. of motion the same in both reference frames
=> <u>Invariant</u> with respect to Galilean transformations

However, problem: if u=c => we get velocity $v_x = c + v'_x > c$ which contradicts the second principle

The assumption t =t’ (concept of simultaneity) has to be revised

2. Relativity of Simultaneity

- Measuring times and time intervals involves the concept of **simultaneity.**
- In a given frame of reference, an **event** is an occurrence that has a definite position and time. When you say that you awoke at seven o'clock, you mean that two events (your awakening and your clock showing 7:00) occurred *simultaneously.*
- The **fundamental problem** in measuring time intervals is this: In general, two events that are simultaneous in one frame of reference are *not* simultaneous in a second frame that is moving relative to the first, even if both are inertial frames.

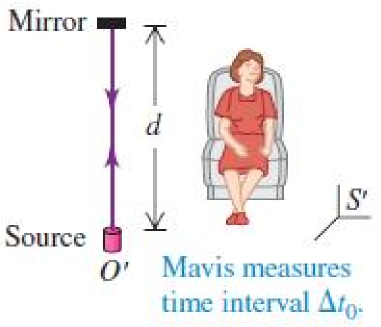
3. Relativity of Time Intervals

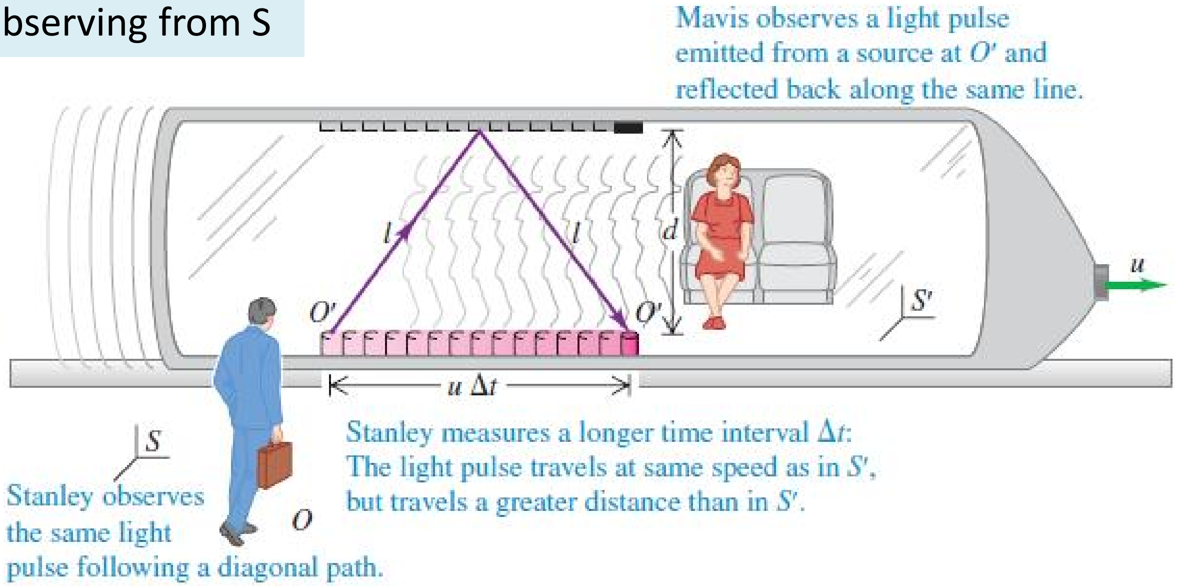
=> derive a quantitative relationship between time intervals in different coordinate systems.

Assume an experiment (light pulse/mirror) in S' moving with u<c with respect to S

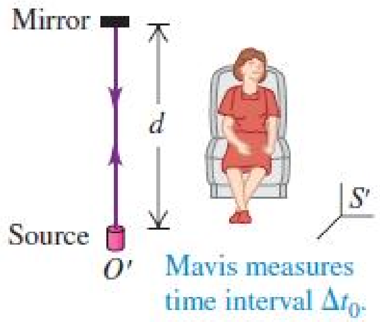
Observing from S'

Observing from S

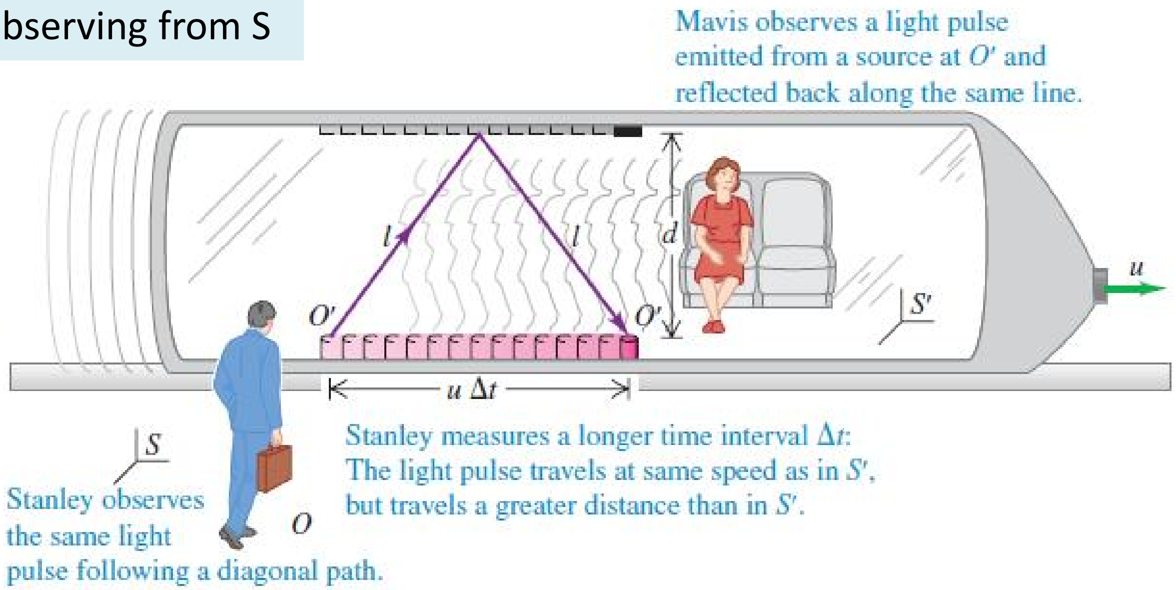




Observing from S’



Observing from S



The observation time interval

$$\Delta t_0 = \frac{2d}{c}$$

$$\Delta t = \frac{2l}{c}$$ With $$l = \sqrt{d^2 + \left(\frac{u\,\Delta t}{2}\right)^2}$$

Substituting d we get a relation between Δt and Δt_0

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - u^2/c^2}}$$

Because u<c

=> $\Delta t > \Delta t_0$

Time dilation

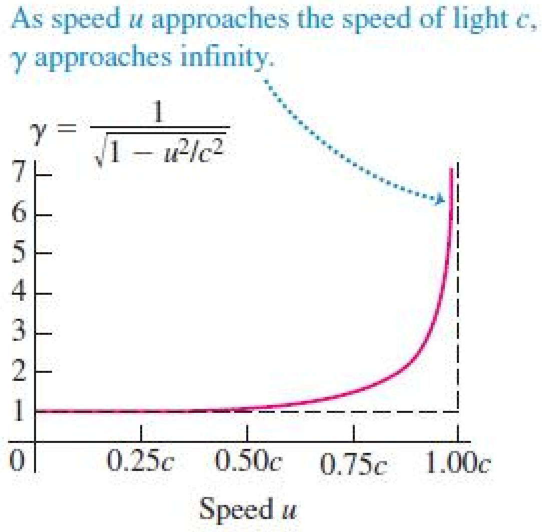
$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}}$$

Lorentz factor
(relativistic correction)

$$\Delta t = \gamma \, \Delta t_0 \qquad \text{(time dilation)}$$

- ❑ ¥**>>1** => relativistic regime
- ❑ ¥¼**1** (u<<c) => nonrelativistic regime

-moving electrons (at v¼10^7 m/s)
-Photons (u=c)
- Particles in accelerators
have to be treated as relativistic



Time dilation at 0.990c

*High-energy subatomic particles coming from space interact with atoms in the earth's upper atmosphere, in some cases producing unstable particles called **muons**. A muon decays into other particles with a mean lifetime of 2.20«s, as measured in a reference frame in which it is at rest. If a muon is moving at 0.990c relative to the earth, what will an observer on earth measure its mean lifetime to be?*

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - u^2/c^2}} = \frac{2.20\ \mu\text{s}}{\sqrt{1 - (0.990)^2}} = 15.6\ \mu\text{s}$$

mean lifetime of the muon in the earth frame is about seven times longer than in the muon's frame

- ❑ verified experimentally;
- ❑ first experimental confirmation of the time dilation formula.

The Twin Paradox

Consider identical twin astronauts named Eartha and Astrid.

- ❑ Eartha remains on Earth while her twin Astrid takes off on a high-speed trip through the galaxy.

- ❑ Because of time dilation, Eartha observes Astrid's heartbeat and all other life processes proceeding more slowly than her own. Thus to Eartha, Astrid ages more slowly; when Astrid returns to earth she is younger (has aged less) than Eartha.

- ❑ **Paradox**: All inertial frames are equivalent. *Can't Astrid make exactly the same arguments to conclude that Eartha is in fact the younger?* Then each twin measures the other to be younger when they're back together, and that's a paradox.

- ❑ **Paradox resolved** : the twins are not identical in all respects. While Eartha remains in an approximately inertial frame at all times, Astrid must accelerate with respect to that inertial frame during parts of her trip in order to leave, turn around, and return to earth.
  Eartha's reference frame is always approximately inertial; Astrid's is often far from inertial.
  => *real physical difference between the circumstances of the two twins. Careful analysis shows that Eartha is correct; when Astrid returns, she is younger than Eartha.*

4. Relativity of Length

Not only does the time interval between two events depend on the observer's frame of reference, but the *distance* between two points may also depend on the observer's frame of reference.

! The concept of simultaneity is involved

Lengths Parallel to the Relative Motion

$$l = l_0\sqrt{1 - \frac{u^2}{c^2}} = \frac{l_0}{\gamma}$$

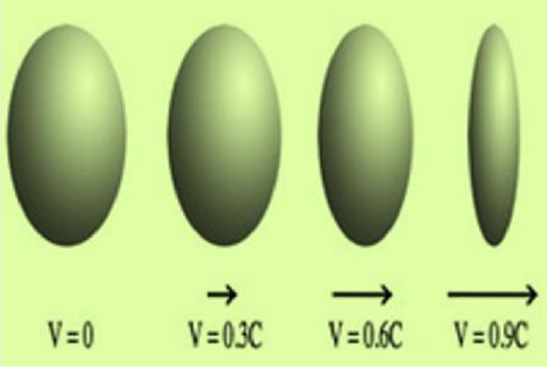
l_0 = measured length of the object at rest
l = measured length of the moving object

Length contraction: the length measured in S in which the ruler is moving, is shorter than the length measured in its rest frame S'

Lengths perpendicular to the Relative Motion

Lengths that are measured perpendicular to the direction of motion are not contracted.

How an Object Moving Near c Would Appear



5. The Lorentz Transformations

- Classical Galilean coordinate transformation equations assume t=t'
- Gave non valid description in the relativistic regime

$$x' = \frac{x - ut}{\sqrt{1 - u^2/c^2}} = \gamma(x - ut)$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - ux/c^2}{\sqrt{1 - u^2/c^2}} = \gamma(t - ux/c^2)$$

(Lorentz coordinate transformation)

Lorentz coordinate transformation

=

relativistic generalization of the Galilean coordinate transformation

- Space and time have become intertwined; we can no longer say that length and time have absolute meanings independent of the frame of reference.

- time + 3 dimensions of space collectively as a four-dimensional entity called **spacetime**, and we call together the **spacetime coordinates of an event**.

The Lorentz Velocity Transformation

$$v'_x = \frac{v_x - u}{1 - uv_x/c^2}$$

$$v_x = \frac{v'_x + u}{1 + uv'_x/c^2}$$

When u<<c we get in the classical (Galilean) regime

...some more Maths

In the theory of relativity, a four-vector or 4-vector is a vector in Minkowski space, a four-dimensional real vector space.

$$x^a := (ct, x, y, z)$$ 4-vector

Minkowski spacetime

Minkowski metric

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2.$$

Euclidean metric

$$ds^2 = dx^2 + dy^2 + dz^2$$

Euclidian 3D space

3 vector x^α: (x,y,z)

Flat spaces
Obey Pythagoras theorem

Basis of special relativity, valid for absence/low gravity

The Lorentz Transformations

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \gamma ct - \gamma\beta x \\ \gamma x - \beta\gamma ct \\ y \\ z \end{pmatrix}.$$

$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}}$$

$$= u / c$$

Gravity -> curved space ->non-Euclidian geometry

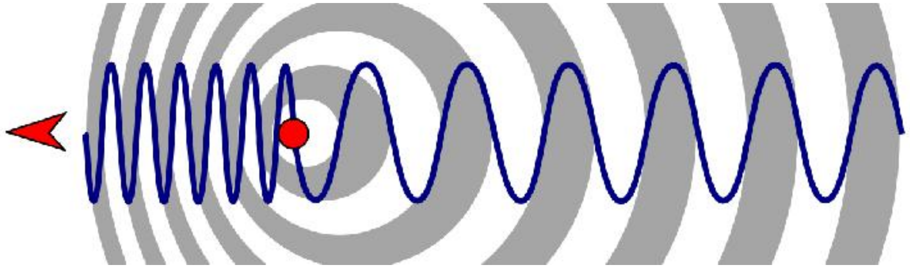
6. The Doppler Effect for Electromagnetic Waves

EM wave move with v=c
=> Relativistic corrections

❑ Classical Doppler effect

$$f = f_0 \frac{1}{1 \mp \frac{v_s}{u}}$$

(- approaching source, + source moving away



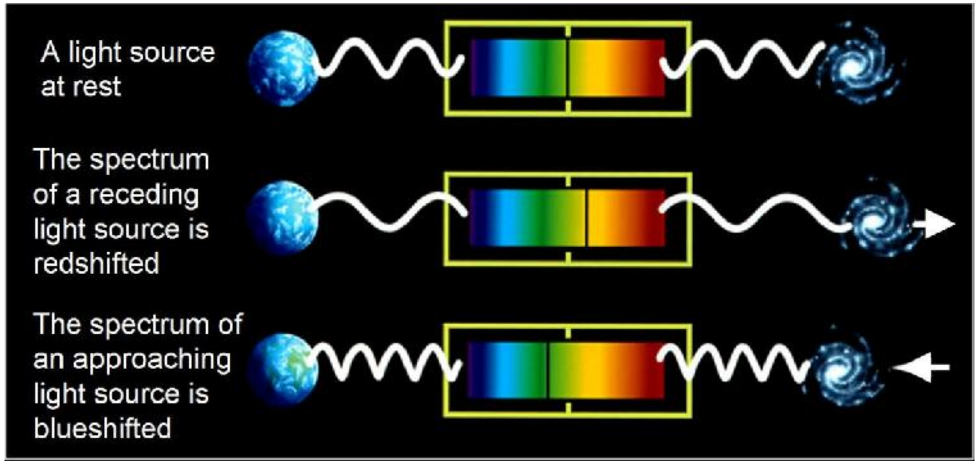
❑ Relativistic Doppler effect

$$f = \sqrt{\frac{c+u}{c-u}} f_0$$

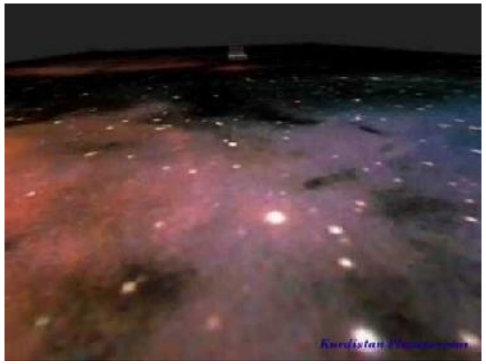
source approaching observer

$$f = \sqrt{\frac{c-u}{c+u}} f_0$$

source moving away from observer



radar gun
Δf+beats=>u



Red-Shift of radiation emmited by galaxies observed => Universe in expansion

7. Relativistic Momentum

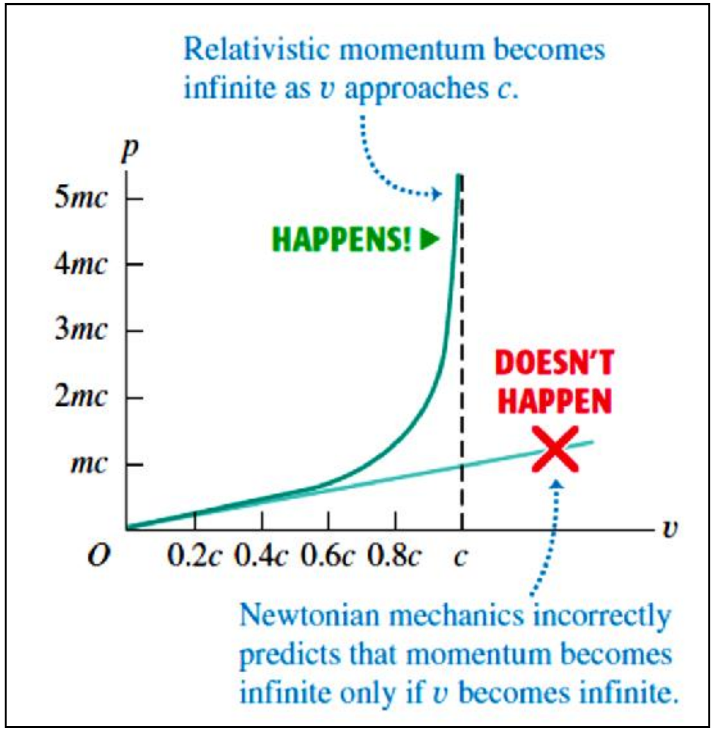
- Newton's laws of motion have the same form in all inertial frames of reference.
- When using transformations to change from one inertial frame to another, the laws should be invariant (unchanging).
- The relativistic regime requires corresponding generalizations in the laws of motion and the definitions of momentum and energy.

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}} \qquad \text{(relativistic momentum)}$$

$$\vec{p} = \gamma m \vec{v}$$

m= the rest mass

- *material particle*= particle that has a nonzero rest mass
- *photon* (light quanta) = zero rest mass



Relativity, Newton's Second Law

Newtonian mechanics

$$\vec{F} = \frac{d\vec{p}}{dt}$$

Relativistic mechanics

$$\vec{F} = \frac{d}{dt}\frac{m\vec{v}}{\sqrt{1 - v^2/c^2}}$$

$$F = \frac{m}{(1 - v^2/c^2)^{3/2}}a \qquad (\vec{F} \text{ and } \vec{v} \text{ along the same line})$$

$$F = \gamma^3 ma$$

$$a = \frac{F}{m}\left(1 - \frac{v^2}{c^2}\right)^{3/2}$$

constant force does not cause constant acceleration
as a particle's speed increases, the acceleration caused by a given force continuously *decreases*
V->C => a->0 no matter how big would be F !!!

Relativistic Mass

From

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}}$$

One can denote

$$m_{\text{rel}} = \frac{m}{\sqrt{1 - v^2/c^2}}$$

"relativistic mass"

a rapidly moving particle undergoes an increase in mass

8. Relativistic Work and Energy

Relativistic Kinetic Energy

When the net force an displacement are in the same direction

$$W = \int_{x_1}^{x_2} F\,dx = \int_{x_1}^{x_2} \frac{ma\,dx}{(1 - v^2/c^2)^{3/2}}$$

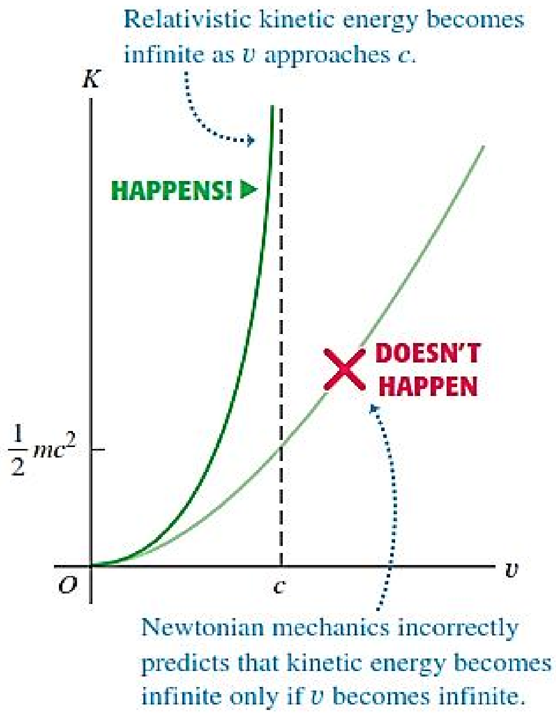
$$a\,dx = \frac{dv_x}{dt}dx = dx\frac{dv_x}{dt} = \frac{dx}{dt}dv_x = v_x dv_x$$

Kinetic energy = work done on a particle to move it from rest to the speed v

$$K = W = \int_0^v \frac{mv_x dv_x}{(1 - v_x{}^2/c^2)^{3/2}}$$

$$K = \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2 = (\gamma - 1)mc^2$$

Relativistic kinetic energy



Rest energy and E=mc²

From

$$K = \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2 = (\gamma - 1)mc^2$$

= difference between some **total energy E** and an energy mc2 that is even in rest

Term depending on the motion

Term independent on the motion = = **rest energy of the particle**

$$E = K + mc^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}} = \gamma mc^2$$

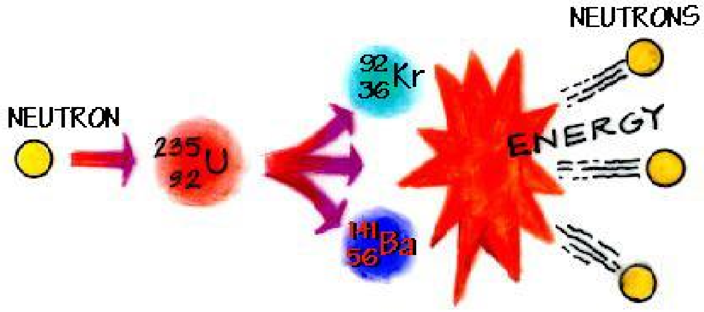
$$E = m_{rel}c^2$$

Rest energy = energy associated with the rest mass m

Mass-energy conversion

fundamental principle involved in the generation of power through nuclear reactions.

When a uranium nucleus undergoes fission in a nuclear reactor, the sum of the rest masses of the resulting fragments is *less than the rest mass of the parent nucleus. An amount of* energy is released that equals the mass decrease multiplied by c².



Most of this energy can be used to produce steam to operate turbines for electric power generators.

Relationship total energy-momentum

$$E^2 = (mc^2)^2 + (pc)^2$$

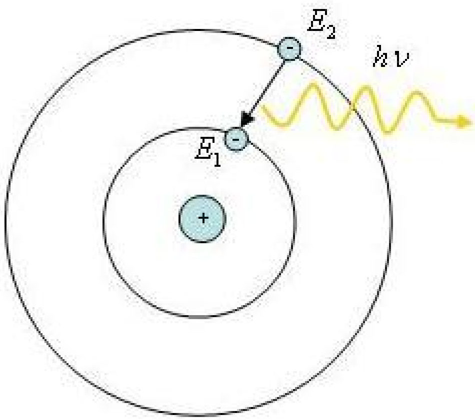
a particle may have energy and momentum even if it has no rest mass

$m=0 \quad => \quad \boldsymbol{E=pc}$

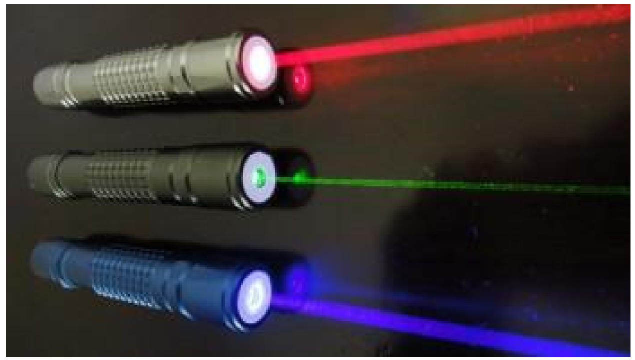
Zero mass particles always travel with the speed of light in vacuum

- *Photons* = quantum of electromagnetic radiation

Quantum mechanics = photons emitted when electron "falls" from E_2 to E_1 ($E_2>E_1$)



LED



LASER diodes

=> optoelectronics

9. Newtonian Mechanics and Relativity

the Newtonian formulation is still **accurate** whenever speeds are small in comparison with the speed of light in vacuum

V<<C

time dilation, length contraction, and the modifications of the laws of motion are so small that they are unobservable

The laws of Newtonian mechanics are:

- not *wrong* BUT *incomplete*
- a limiting case of relativistic mechanics
- *approximately correct when v<<c, exactly correct* when all speeds approach zero.

Relativity does not completely destroy the laws of Newtonian mechanics but *generalizes them.*

Common pattern in the development of physical theory

- Whenever a new theory is in partial conflict with an older, established theory, the new must yield the same predictions as the old in areas in which the old theory is supported by experimental evidence.
- Every new physical theory must pass this test, called the **correspondence principle**

Old theory = limit case of newer theory

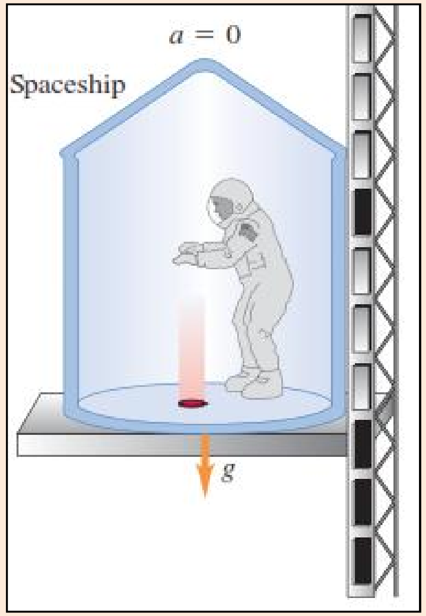
The General Theory of Relativity

Question: does the special theory of relativity gives the final word on mechanics or whether *further generalizations are possible or necessary?*

Example (virtual experiment):

An astronaut is about to drop her watch in a spaceship.

(1) **In gravity-free space**, the floor accelerates upward at *a=g and hits the watch.*

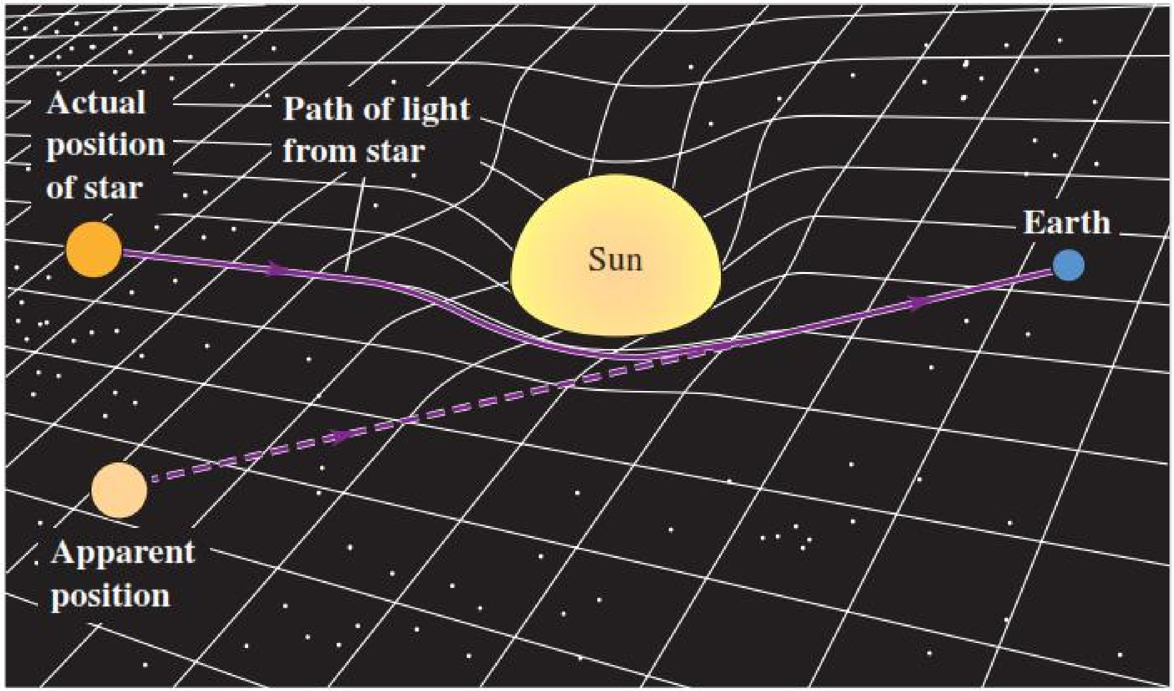


(2) On the earth's surface, the watch accelerates downward at *a=g and hits the* floor.

Without information from outside the spaceship, the astronaut cannot distinguish situation (1) from situation (2).

Basis of Einstein's **general theory of relativity.**
If we cannot distinguish experimentally between a uniform gravitational field at a particular location and a uniformly accelerated reference frame, then there cannot be any real distinction between the two.

- we may represent *any gravitational field in terms of special characteristics of the coordinate* system.
- This require even more sweeping revisions of our spacetime concepts than did the special theory of relativity.
- In the **general theory of relativity** the geometric properties of space are affected by the presence of matter

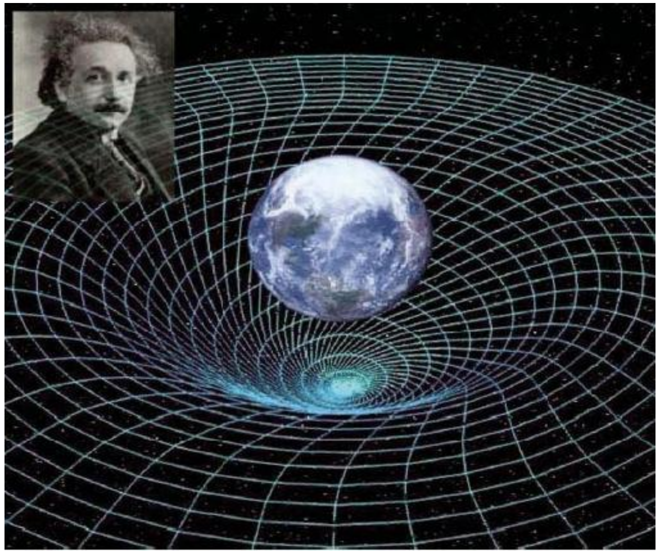


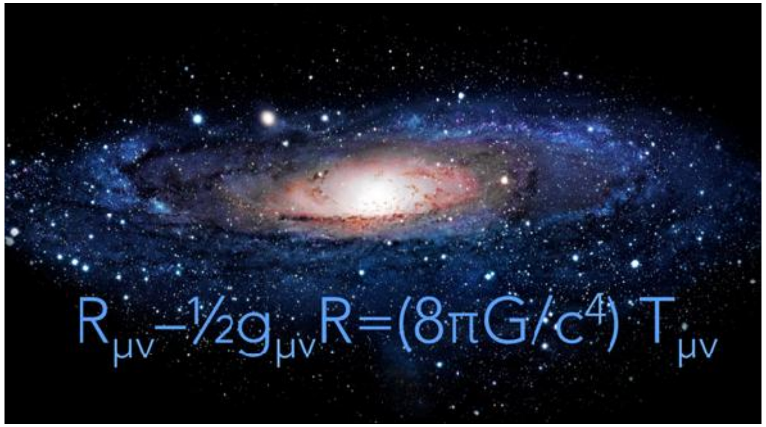
General relativity /geometric theory of gravitation

the space (a plane) as being distorted as shown by a massive object (the sun). Light from a distant star (solid line) follows the distorted surface on its way to the earth. The dashed line shows the direction from which the light appears to be coming. The effect is greatly exaggerated; for the sun, the maximum deviation is only 0.00048°.

General relativity or the geometric theory of gravitation

- provides a unified description of gravity as a geometric property of space and time, or spacetime.
- In particular, the curvature of spacetime is directly related to the energy and momentum of whatever matter and radiation are present.
- The relation is specified by the Einstein field equations, a system of partial differential equations.





Einstein's theory has important astrophysical implications

it implies the existence of black holes—regions of space in which space and time are distorted in such a way that nothing, not even light, can escape—as an end-state for massive stars.

General relativity or the geometric theory of gravitation

- Albert Einstein's general theory of relativity is one of the towering achievements of 20th-century physics. Published in 1916, it explains that what we perceive as the force of gravity in fact arises from the curvature of space and time.

- Einstein proposed that objects such as the sun and the Earth change this geometry. In the presence of matter and energy it can evolve, stretch and warp, forming ridges, mountains and valleys that cause bodies moving through it to zigzag and curve. So although Earth appears to be pulled towards the sun by gravity, there is no such force. It is simply the geometry of space-time around the sun telling Earth how to move.

- The general theory of relativity has far-reaching consequences. It not only explains the motion of the planets; it can also describe the history and expansion of the universe, the physics of black holes and the bending of light from distant stars and galaxies.

- Matter causes space to curve. It is posited that gravitation is not a force, as understood by Newtonian physics, but a curved *field* (an area of space under the influence of a force) in the space-time continuum that is actually created by the presence of mass.

Relativity and causality

- In Einstein's Special Relativity, it is established that information could not be transmitted faster than the speed of light in vacuum, the so called "cosmic speed limit".

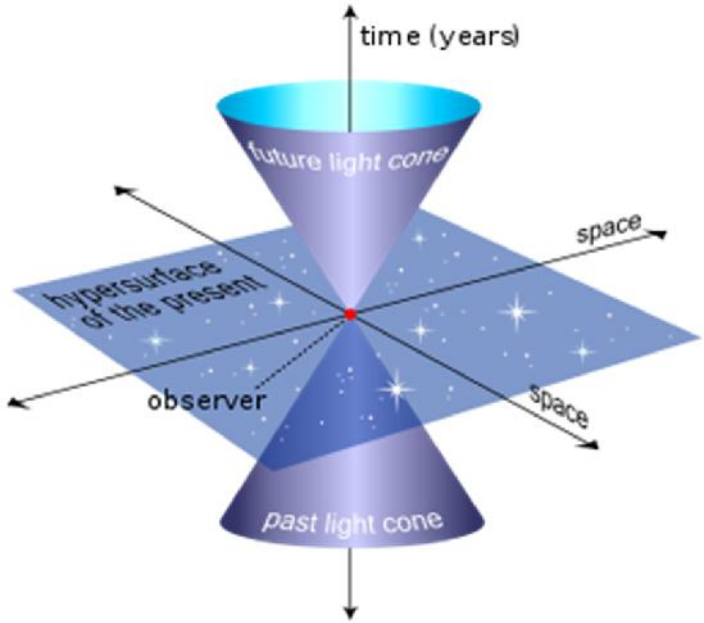
- The limit imposed by the speed of light on information travel divides the surrounding spacetime into causally distinct regions.

- For a given object, A, the present is its locus, a single point in spacetime at the vertex of a light cone.

- **A light cone** is the conic consisting of all events that can be connected to A via light rays.

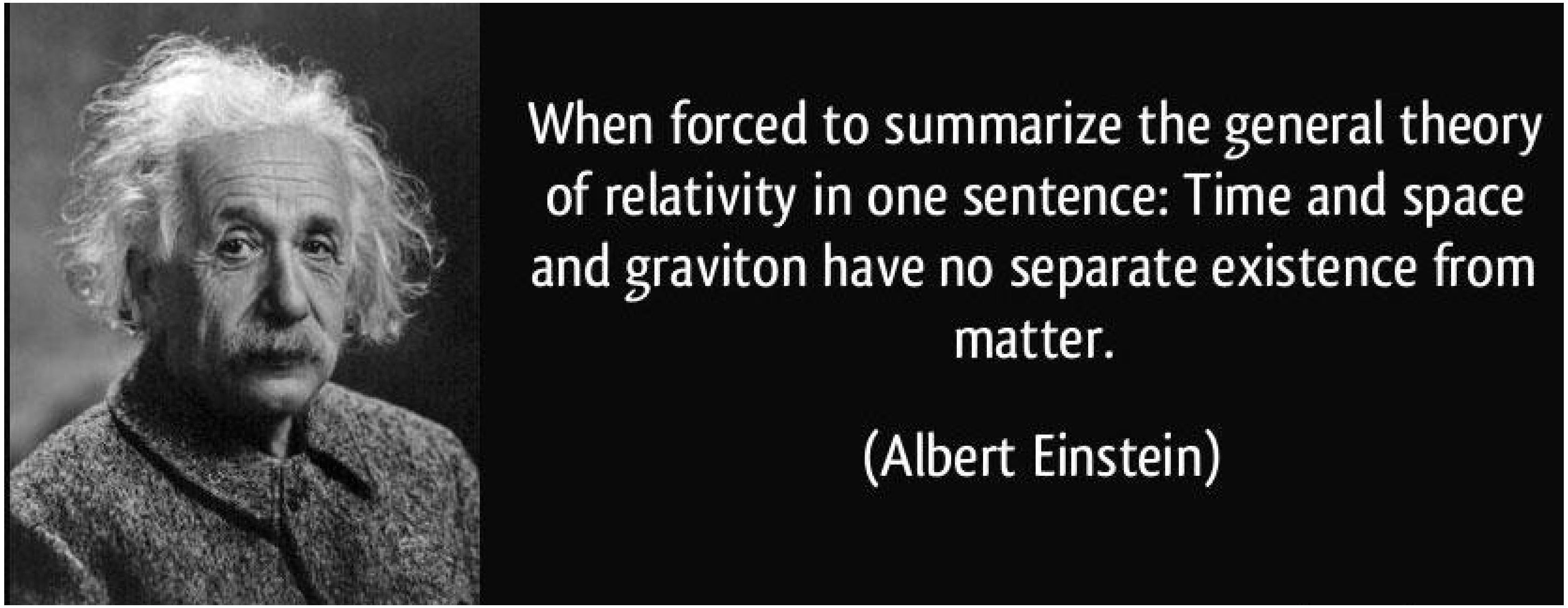
Novelty is possible because the future is not pre-decided. This diagram represents a moment of concrescence. The "future light cone" is filled with possibilities, but they are not yet actualized. Some may be actualized; some not. Everything depends on the "decision" in the present moment.

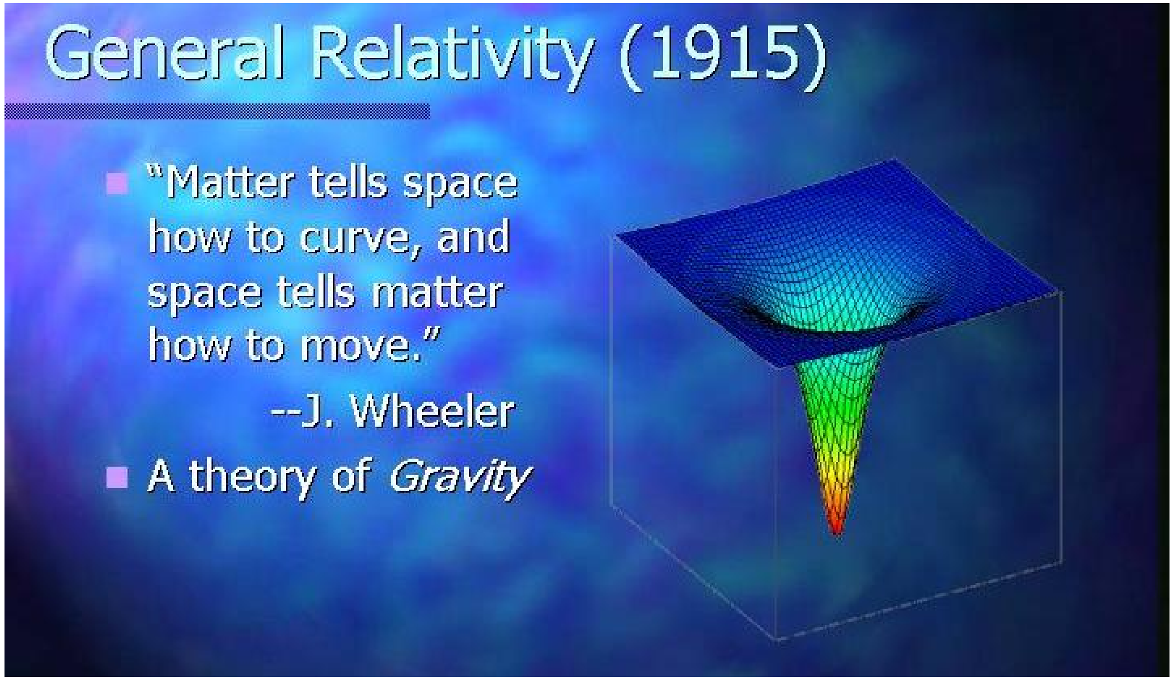
An example of a light cone, the three-dimensional surface of all possible light rays arriving at and departing from a point in spacetime.



- Events falling within the past light cone can reach or exert causal influence on A.

- Events falling within the future light cone can in turn be influenced by the A. Events outside of A's lightcone cannot be causally linked to A, because in order to do so, information has to be transmitted faster than the cosmic speed limit. When we look at the Universe we are actually seeing just our past light cone in all of spacetime


When forced to summarize the general theory
of relativity in one sentence: Time and space
and graviton have no separate existence from
matter.
(Albert Einstein)


General Relativity (1915)
"Matter tells space how to curve, and space tells matter how to move."
--J. Wheeler
A theory of *Gravity*